Monopolistic Screening with Buyers Who Sample

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- Computational constraints make it impossible for them to have access and evaluate all available alternatives
- This impacts how sellers determine their product line and pricing decisions

Goal

- Study a problem of product line design with informational frictions
- Build a simple model that
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What I do

$\Rightarrow$ I propose a model in which buyers cannot evaluate all available alternatives presented by the seller

$\Rightarrow$ Instead, they only sample some of the alternatives

$\Rightarrow$ The main question is how the optimal menu/mechanism looks like here

Spoiler alert!

Model:	Mussa and Rosen (1978)	+	boundedly rational buyers
			$\downarrow$
			unknown menu + sampling

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All results are up to measure zero cases

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$\rightarrow$ If the buyer rejects the offer then both get zero

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- Instead they will sample offers from the menu uniformly at random
- The number of samples is fixed
- Outside option $(0, 0)$ is always available for consumers
- Since duplicating all offers makes no difference, I focus on menus with minimum size

Timing

Seller designs menu

$\downarrow$

Buyers sample from the menu

$\downarrow$

Buyers decide whether to accept
one of the sampled offers

Seller's problem with a single sample

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- This implies that only "participation" constraints are relevant

Main result with a single sample

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Consider the single sample problem with two valuations. The optimal menu includes a single offer.

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Consider the single sample problem with two valuations. The optimal menu includes a single offer.

- Hence, in an environment with a single sample, the *effective variety* offered is reduced

Sketch of proof:

- Step 1: only "efficient" offers are included in the menu: (θ, θ^2) for some θ
    - No incentive compatibility constraints since only single offer is observed each time
    - If offer with quality q is drawn, for which last type accepting is θ, optimal to price it at $p = \theta q$
    - Then, if offer is accepted by $\theta' \geq \theta$, optimal to match efficient quality provision for θ

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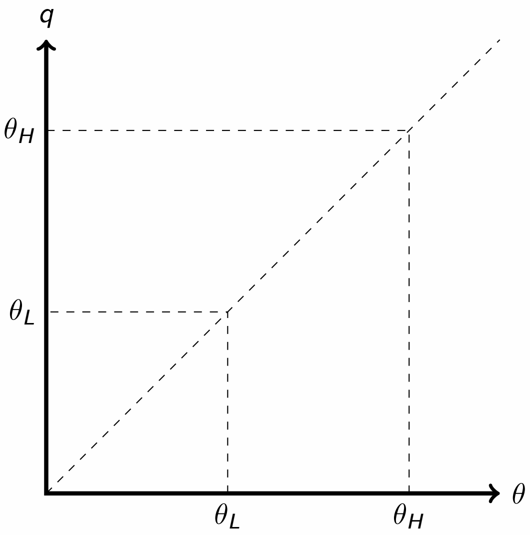
- Step 2: given that only offers of this form are offered optimal menu is determined by a linear problem
  - Solution involves assigning all mass to "best" offer only

Optimal menu with a single sample (in pictures)



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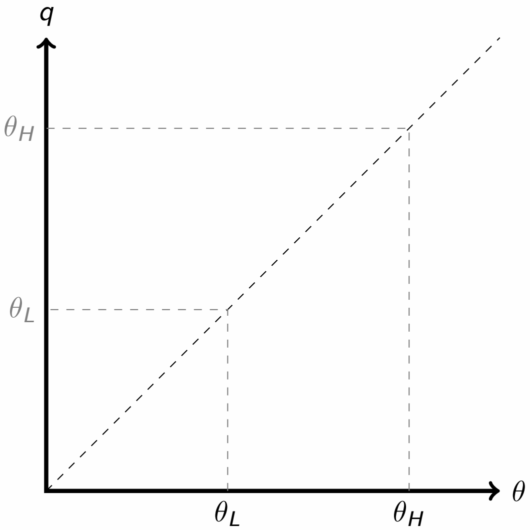

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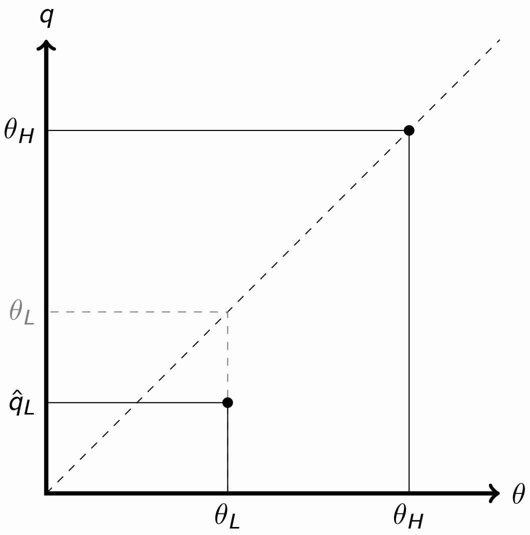

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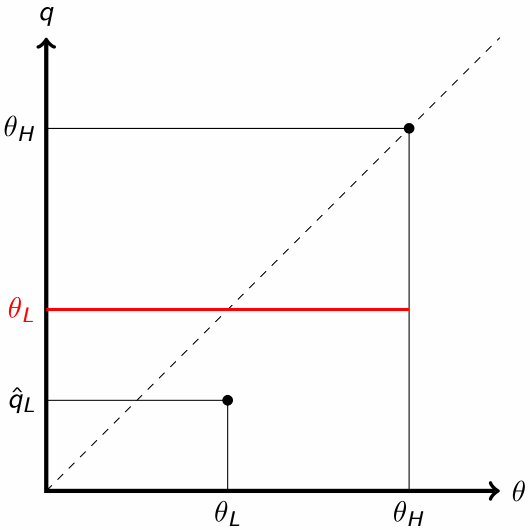

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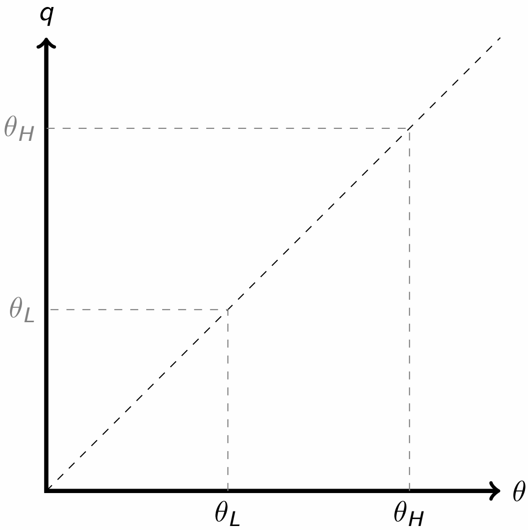

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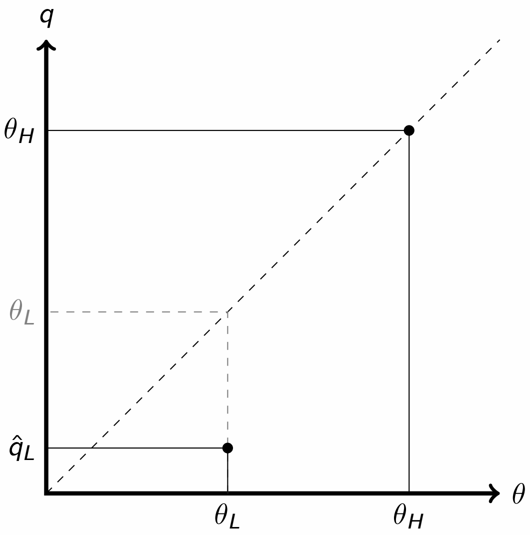

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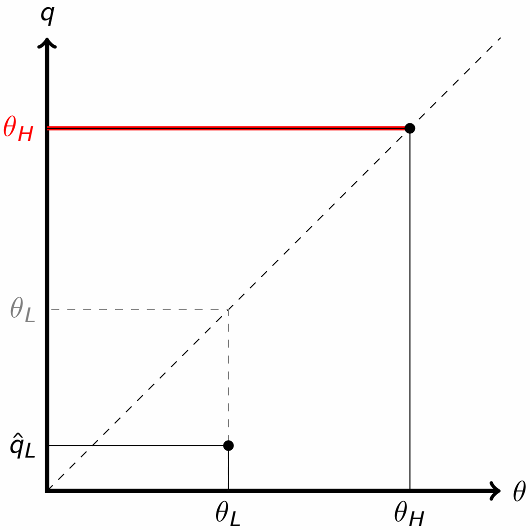

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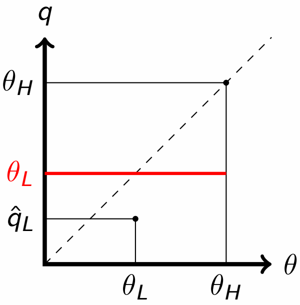
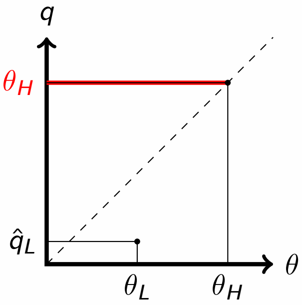


(a) $\alpha < \hat{\alpha}$: optimal menu offers only $q = \theta_L$ (red). All buyers accept the offer.



(b) $\alpha > \hat{\alpha}$: optimal menu offers only $q = \theta_H$ (red). Only buyers with valuation θ_H accept the offer.

Seller's problem with two samples

- For a menu of size m, buyers will observe a single offer i with probability $1/m^2$, and two offers j and k with probability $2/m^2$
- Since more than one alternative would be evaluated with positive probability (unless all offers are identical), there would be relevant incentive compatibility constraints to satisfy now
- This makes the characterization of the optimal menu challenging
- To guarantee existence, I assume that there is a limit M on the size of the menu the seller could design, and consider the case in which M is large

Results with two samples

Lemma

Consider the problem with two samples. Suppose that the optimal menu contains only two offers (q_a, p_a) *and* (q_b, p_b)*. Then, for* M *large enough, the expected profits from menus* $\{(q_a, p_a)\}$ *and* $\{(q_b, p_b)\}$ *must be the same.*

$$\Downarrow$$

Proposition

Consider the problem with two samples and two valuations. Suppose M *is large enough, Then, the optimal menu does not contain only two offers.*

Intuition behind Lemma

- Fix (q_a, p_a) and (q_b, p_b)
- Let R_i the value generated for the seller if buyers observe $i = a, b, ab$
- Let x the probability **a** is drawn
- Consider the following problem for the seller

$$\max_x x^2 R_a + (1-x)^2 R_b + 2x(1-x) R_{ab}$$

- If exists, the interior solution is

$$x^* = \frac{1}{1 + \frac{R_{ab} - R_b}{R_{ab} - R_a}}$$

- Note, $x^* = 1/2 \iff R_a = R_b$

Intuition behind Lemma

- A necessary condition is $R_{ab} > \max\{R_a, R_b\}$ (i.e., there must be gains from using a menu)
- Assume $R_a \geq R_b$
- Starting from a menu only containing **a**, including **b** induces...

$$\begin{array}{rl} \text{“Gain”} & R_a \rightarrow R_{ab} \\ \text{“Loss”} & R_a \rightarrow R_b \end{array}$$

- x^* balances this tradeoff
  - If **b** drawn with small probability ϵ, more likely to observe $\{a, b\}$ instead of **b** only $\Rightarrow$ overall gain from including **b**
  - If $R_a = R_b$, no cost of including **b**, so optimal to maximize prob. of $\{a, b\}$
  - If $R_a > R_b$, then costly to include **b** and having both with same probability is too costly $\Rightarrow$ optimal to "bias" toward **a**

From Lemma to Proposition and beyond

- For two valuations and two samples, I could show that never optimal to set **a** and **b** such that $R_a = R_b$ (up to a very specific set of parameters)

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- Extending the proposition to a more general structure is still work in progress

Heterogeneity in sample sizes

- What if there are consumers with one and two samples at the same time?
- It can be shown that the problem is qualitatively similar to the case in which all consumers have two samples
- Hence there is little loss on considering all consumers having the same sample size

Extension: Submenus with a single sample

- Consider the possibility of offering small menus instead of single alternatives on each draw.
- A mechanism is now a collection of (sub)menus of quality-price pairs.
- Each submenu has a limited size S.
- Valuations are distributed over an interval $[\theta_L, \theta_H]$ according to some distribution F.
- We consider the case in which buyers sample only once.

Extension: Submenus with a single sample

Proposition

Consider the environment with finite-size submenus and a single sample. Suppose Assumption 1 holds. Then, the optimal mechanism uses a single submenu.

- Same intuition as in main theorem:
  - No IC implies each submenu must be optimal given submenu's size
  - Resulting problem is again a linear problem
  - Hence, solution involves maximizing the probability of the best option (i.e., submenu).

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Assumption 1: The optimal mechanism under full-consideration and a menu of size up to S is unique.

Related literature

- Product line design and pricing: Mussa and Rosen (1978), Villas-Boas (2004), Doval and Skreta (2022), Garrett et al (2019), Sandmann (2023)
- Revenue maximization with samples: Dhangwatnotai et al (2015), Babaioff et al (2018), Daskalakis and Zampetakis (2020), Fu et al (2021)
- Sampling/$S(1)$ equilibrium: Osborne and Rubinstein (1998, 2003), Spiegler (2006), García-Echeverri (2021)
- Search: Weitzman (1979), Burdett and Judd (1983), Doval (2018), Ursu et al (2021), Safonov (2022), Fershtman and Pavan (2022)

Concluding remarks

- I presented a model in which a seller interact with boundedly rational buyers which cannot observe the menu designed by her and instead get samples from it
- I showed that the optimal menu when buyers have access to a single sample involves including a single offer, matching the best contract for one type of buyers
- In the case of two samples, I showed that the optimal menu cannot contain only two alternatives, each sampled with probability $1/2$

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What is next?

$\Rightarrow$ Full characterization for more than two samples

$\Rightarrow$ Study the effect of competition on the seller's problem

$\Rightarrow$ Allow the seller to use targeted menus/ads

$\Rightarrow$ Applications: taxes and social insurance systems

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Thanks!

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