# Monopolistic Screening with Buyers Who Sample*

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April, 2023

EXTREMELY PRELIMINARY AND INCOMPLETE

**Abstract**

We study a monopolistic screening problem with boundedly rational buyers and a noisy communication technology. In our model the seller designs a menu of quality-price pairs to a continuum of buyers that remain unaware of the offered menu. Instead, buyers have access only to a finite number of samples from the menu offered by the seller and then decide which sampled alternative to purchase if any. This procedure give arise to random consideration sets from the perspective of the buyers. We show that if there is a single sample available, the seller will optimally choose to offer a single alternative, while if two samples are available then neither offering a single alternative nor two alternatives is necessarily optimal.

## 1 Introduction

Consumers receive many offers for buying a huge amount of different products and brands. However, time and cognitive constraints (such as limited attention or choice overload) could make the process of evaluating all available alternatives difficult. Moreover, from a theoretical perspective, even conjecturing what would or could be offered may be difficult in several markets. This type of limitations would certainly impact the behavior of agents in several market interactions. Given this, it is natural to think that sellers will take this limitations into account when designing the types of products they will be offering.

*I thank Luca Rigotti and Richard Van Weelden for providing guidance in this project. I also thank Nicolas Figueroa and participants of the Pittsburgh Economic Medley Conference for useful comments and suggestions.

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Here we study this problem from the perspective of a monopolist facing boundedly rational buyers. We consider a product design or nonlinear pricing problem as in Mussa and Rosen (1978) but where there is a direct tradeoff between increasing the pool of varieties offered to buyers and the ability to actually screen different types of buyers as having more alternatives makes them less likely to evaluate any particular offer. In our model, buyers are unable to evaluate all alternatives offered by the seller and instead they will be able to evaluate only *samples* from the complete menu offered by the seller. In turn, the seller will take these limitations into account when designing the menu of alternatives she will offer to the buyers.

In particular, we consider the problem of a monopolist (seller) with quadratic costs producing a single good with differentiated quality. Buyers have quasilinear preferences over quality and transfers, and both sides of the market are risk neutral. Buyers can only observe a subset of the varieties offered by the seller and are boundedly rational in the sense that they cannot conjecture what a seller will include in an optimal menu. Hence, they will be able to evaluate only alternatives they could directly observe. We will refer to the process in which buyers observe alternatives as *sampling*.

We will assume that the number of samples a buyer has available is exogenous and fixed, this differences our model from a standard search model and is coherent with the notion of bounded rationality we consider here. While our model is not a traditional search model, it could be interpreted as a model of *passive search* (Ursu et al. (2021)) where the search process cannot be fully controlled by the seller nor the buyer.

Another main difference with work on search models like Weitzman (1979) and Doval (2018) is that we focus on the seller's design problem instead taking the searching behavior of consumers as given, instead of characterizing the search behavior assuming perfect rationality. This also makes our model different from Safonov (2022) which considers a model with a boundedly rational agent which search over a fixed menu, studying his behavior in terms of computational complexity.

We show that when the buyer has a single sample, the seller will optimally choose to offer a single alternative, reducing the variety of alternatives offered compared to the case of perfect observability.

For the case of two samples, we show that the optimal menu for the seller could contain more than two alternative if sampling is with replacement and uniform (i.e., when each alternative is drawn with probability $1/m$ where $m$ is the size of the menu).

In both cases, the cost of introducing more alternatives makes less attractive introducing such. In the case of one sample, this lead to the seller to offer a single version of his product. In the case of two sample, this introduce benefits of having some alternatives drawn more often than others, which makes having only two contracts never optimal.

## 1.1 Related literature

Our model is closely related to the concept of $S(1)$ equilibrium in Osborne and Rubinstein (1998) and Spiegler (2006), and the related idea of sampling equilibrium in Osborne and Rubinstein (2003) and García-Echeverri (2021). One difference is that in our model alternatives available to the buyers are endogenous since they are explicitly designed by the seller, and that at the same time such alternatives are unknown to the buyers while the payoff generated by each of them is known after an alternative is known to be available. Instead in Osborne and Rubinstein (1998), the alternatives are known but their payoff are unknown due to dependence on the behavior of other players, hence sampling is performed in order to learn about payoffs while in our model it is used to learn about the alternatives available.

Spiegler (2006) uses the $S(1)$ procedure in Osborne and Rubinstein (1998) to study a model of competition with boundedly rational buyers. Our sampling model differs from the notion used on those paper and is more in line with the search procedure by Weitzman (1979), in the sense that (random) alternatives must be evaluated and either accepted or not afterwards, instead of obtaining a new draw from the distributions. One main difference is that the two modeling approaches poses is that in their model distributions over different prices and price dispersion arise naturally since there are direct incentives to obfuscate while here it is not clear whether the seller would like to reduce variety or increase it as response to buyers' boundedly rational behavior. Moreover, we focus on a model without competition.

Since the seller could influence the choices of the buyers by choosing different presentations of the menu in the interaction with boundedly rational buyers, our model is also related to the problem of screening when the framing of the alternatives influence the final actions performed by consumers as in Ostrizek and Shishkin (2019) and Salant and Siegel (2018). However, in our model the seller cannot artificially influence the value of the different products she is offering, being allowed only to indirectly influence the consideration sets the buyers would have.

The sampling technology used here give arise to random consideration sets for the buyers as in the problem studied by Fershtman and Pavan (2022). However, in our model these random consideration sets are only "partially" endogenous since they are controlled by the seller but imperfectly since she has no direct control over the sampling process performed by the buyers. Moreover, we focus on the design problem for the seller taking as given the behavior of the buyers while Fershtman and Pavan (2022) focus on the optimal behavior of the agents for a fixed environment. Another difference is that their environment is dynamic and agents are fully rational, while our model is static and has boundedly-rational buyers. Bounded-rationality is also present in the model studied by Safonov (2022), but as Fershtman and Pavan (2022), it focus on the performance of the agents for a fixed setting instead of the design problem faced by the seller as in our model.

Ursu et al. (2021) studies search in the context of online advertising. They propose a model with active and passive search, and present evidence that both types of search are empirically relevant and different in nature. In particular, they show that the behavior captured in passive search is not consistent with the optimal search policy in Weitzman (1979), and that models that treat all search as active could introduce estimation bias on the value of different alternatives based on the order of search. In this context, our model is consistent with passive search only in the sense that buyers in our models make no decision about the intensity or structure of search and receive only random information through the samples. Also, we focus on the problem of a monopolist while Ursu et al. (2021) focus on a competitive market.

Eliaz and Spiegler (2011) also present a model in which buyers face random consideration sets. Their model focus on the outcome of the competition among two sellers that simultaneously choose which product to offer, from a fixed set, as well as the marketing strategy which determines if its product will enter the consideration set of the buyers. Two main differences is that we focus on the problem of a monopolist and we also allow her to design the characteristics of the different alternatives offered.

Hart and Nisan (2019) and Bergemann et al. (2021) follow different approaches to answer similar questions in two standard mechanism design environments: what is the performance of a mechanism if there is limitations on the size of the menu that the designer could offer? Both papers study this question in the context of the multi-product monopolist (Rochet and Choné (1998), Manelli and Vincent (2007)) in different contexts. Hart and Nisan (2019) studies a model without costs and buyers with additive values, and shows that if there is correlation simple mechanisms could have an arbitrarily poor performance in terms of revenue in this context, in contrast with previous results by Hart and Nisan (2017) which have shown that the same class of simple mechanisms achieved good performance in environments without correlation. Bergemann et al. (2021) also considers a model with buyers with additive values but quadratic costs from side of the seller as in Mussa and Rosen (1978) and Armstrong (1996). They study how limited (i.e., finite) information shapes the form of the optimal mechanism and show that the problem is equivalent to the quantization problem in information theory. In this paper, we study a single dimensional nonlinear pricing problem with limited communication but also include boundedly-rational buyers and noisy communication captured by the random sampling procedure that determines the alternatives observed by each buyer.

Our model is also related to literature in computer science and operation research studying mechanism design problems in which the designer has access to samples about the uncertainty in the model instead of a well defined prior distribution over such uncertainty. Dhangwatnotai et al. (2015) study the problem of revenue maximization when only one sample is available. Building upon the result in Bulow and Klemperer (1996), they show that using a random bid as a reserve price guarantees at least half of the revenue of the optimal mechanism. Fu et al. (2021) studies the problem of surplus extraction with an unknown correlated distribution over valuations and access to samples from the true distribution. They show that full extraction is feasible if possible distributions satisfy the condition in Crémer and McLean

(1988) and a bounded number of samples. For two samples, Babaioff et al. (2018) and Daskalakis and Zampetakis (2020) show that studying the problem is technically challenging and obtain some revenue guarantees. The model presented here depart from this literature by considering sampling from the perspective of consumers instead of the designer or seller and expected profit maximization instead of worst-case guarantees.

### 1.2 Outline

The remaining of the paper is structured as follows. In Section 2 we present the general model and the benchmarks. We present the main result with a single sample in Section 3 while in Section 4 we present the result with two samples. In Section 5 we present an extension in which the seller offers a collection of menus from which the buyer obtains a single sample and then decide what product to purchase. Finally, Section 6 concludes.

## 2 Model

We build upon the traditional nonlinear pricing problem in Mussa and Rosen (1978). There is a seller which produces a vertically differentiated good with quality $q \geq 0$ at cost $c(q) = q^2/2$. There is a continuum of buyers (mass one), with privately known valuations $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L > 0$. We denote by $\alpha \in (0,1)$ the fraction of high valuation ($\theta_H$) buyers.

The seller offers a menu or mechanism which is a collection of quality-price pairs, and her goal is to maximize her expected profits under such mechanism. We will refer to a quality-price pair $(q, p)$ as an offer or product, hence a mechanism or menu will be a collection of such offers or products.

If an offer $(q, p)$ is accepted, the seller obtains profits $p - q^2/2$ while a buyer with valuation $\theta$ obtains $\theta q - p$ from that transaction. If an offer is rejected then both the buyer and the seller get payoff zero from that interaction.

In the standard problem, given a menu, buyers will evaluate each alternative and choose the one that gives them the highest payoff. This requires that buyers actually observe the menu or at least "be aware" (i.e., conjecture) what is offered by the seller in equilibrium. Here instead we will assume that buyers are boundedly rational and cannot observe, evaluate nor conjecture what is in the menu offered by the seller. Instead, they will *sample* uniformly at random from the actual menu offered by the seller, and then decide which sampled alternative to purchase if any. We will assume that the number of samples is exogenous and each buyer has the same number of samples available. Moreover, the seller will not be able to offer personalized menus, and every buyer will face the same menu. Note that as the number of alternatives is unknown for buyers, our model differs from the model of sampling introduced by Osborne and Rubinstein (1998) where agents sample each alternative (at least) once. Unknown actions in asymmetric information problem have been used before by Carroll (2015) in a different context. However, in Carroll (2015) the seller is the one facing unknown actions while buyers (agents in their model) knows exactly the actions they have available. In our model the relation is exactly the opposite: alternatives are unknown to buyers but designed (known) by the seller.

Taking the number of samples fixed instead of allowing buyers to choose how many samples to draw is consistent with our notion of bounded rationality. Allowing buyers to optimally choose the number of samples to draw will require them to compute the expected utility that they will get from adding an extra sample, which requires them to form a prior over the potential contracts that the menu could offer. Our main behavioral assumption here rules out such possibility, since we are assuming that buyers cannot conjecture what would be in the menu the seller offers.

Naturally, this relates our model with model of search but differentiates it from them by fixing the searching strategy of buyers. In that sense, our model is closer to a model of passive search as in Ursu et al. (2021), in which buyers are exposed to offers not because they are actively searching for products as in a standard (active) search model. Ursu et al. (2021) finds that in some online interactions, a passive search model provides a better explanation than a standard model of active search.

We take a partial implementation approach here, breaking any indifferences in favor of the seller. Moreover, since duplicating all alternatives in the menu generates no difference for neither the seller nor the buyers, we will focus only on menus with minimal size. As in Hart and Nisan (2019), we will ignore the outside option $(0, 0)$ as part of the menu since buyers will always be allowed to reject the offers they draw.

We will denote the number of samples by $n$ and assume sampling is with replacement if $n > 1$. That is, the menu designed by the seller is fixed before the buyers interact in the market. This also implies the seller will be unable to target buyers based on the alternatives they have already sampled. Note this assumption is consistent with buyers being able to receive the same result after two searches using different keywords or platforms, or receiving the same quote twice.

We also assume there is an exogenous limit $\overline{M} > 2$ to the maximum size of the menu offered by the seller. While this assumption is not necessary for the results with a single sample, it is requires to guarantee the existence of an optimal menu for two or more samples.

Finally, all results in the following sections are hold up to measure zero set of parameters.

## 2.1 Benchmark

Before diving into the solution to our model, we will review the solution in the standard Mussa and Rosen (1978) setting in which the menu is observed by buyers before deciding which alternative to buy.

We start with the characterization of the efficient allocation in this context. Given the simple structure we have assumed here, the efficient allocation if the menu is observable involve matching the quality provided to each type to his valuation, i.e., quality $q = \theta$ for each type $\theta$.

In the case of profit maximization, the optimal menu under perfect observability could take one of two configurations. Either it contains $(\theta_H, \theta_H^2)$ only or it contains $(\hat{q}_L, \theta_L \hat{q}_L)$ and $\left(\theta_H, \theta_H^2 - (\theta_H - \theta_L)\hat{q}_L\right)$ where $\hat{q}_L = \theta_L - \frac{\alpha}{1-\alpha}\left(\theta_H - \theta_L\right) < \theta_L$. In the first case, only high valuations buyers are served while low valuations buyers are completely excluded. In the second case, two options are offered allowing buyers to self select into their preferred contract. In this case, the high valuation buyers receive the efficient quality level for their valuation and obtain informational rents in order to induce them to reveal their private information, while low valuations buyers receive quality beyond the efficient level and get a payoff of zero. Note that the first case is completely implementable if buyers cannot observe the full menu but the second case is not.

# 3 The problem with a single sample

We now analyze the problem with a single sample. This is the simplest environment we could consider in this setting and is a common starting point in related models (e.g., Osborne and Rubinstein (2003), Spiegler (2006), Dhangwatnotai et al. (2015)).

With a single sample and a given menu $\{(q_1, p_1), ..., (q_m, p_m)\}$ of size $m$, each buyer will observe each alternative with probability $1/m$. Note that introducing a copy of an offer already in the menu has an impact on the perceived menu for the buyers, as it increases the probability of drawing that particular offer and reduces the probability of any other offer as long as there is more than one (different) offer in the menu. This feature is absent in the classic model in which having copies of any offer (or dominated offers) has no impact in the final decision of the buyers nor in the seller's profits.[1] This also implies that while the relative probability of different offers is held constant (due to the assumption of uniform sampling), the seller has (almost) full control on the effective relative probability for each alternative, i.e. the relative probability adjusted by the presence of equivalent offers.

Since, buyers will be able to draw a single offer from the menu there will be no relevant incentive compatibility constraints. That is, a contract $(q, p)$ drawn from the menu is only compared with the outside option of rejecting the offer and obtaining payoff zero. Hence, no other contract in the menu is relevant for this comparison since the buyer never observes nor evaluates more than one offer at a time.

This simplifies the analysis of the case with a single sample and allows us to give a complete characterization of the optimal menu in this case.

**Proposition 1.** *Consider the single sample problem with two valuations. In the optimal*

[1] A similar feature also arise in models of common agency as Martimort and Stole (2002).

*mechanism, the seller always prefer to offer a single contract $(q^*, t^*)$. Moreover, such contract is either accepted only by the high valuation buyers which obtain zero expected utility, or it is accepted by all buyers and only the high valuation buyers obtain a positive payoff. In the first case, efficient quality is provided but only to high valuation buyers, while in the second case only the low valuation buyers receive the efficient quality.*

*Proof.* First, we show that only "efficient" quality-price combinations will be offered in an optimal mechanism.

Suppose offer $(q, p)$ is offered by the seller. There are two possibilities: either $(q, p)$ is accepted by all buyers or only by high valuation buyers due to complementarity between quality and valuations.

1. If only accepted by high valuation buyers, then it is optimal to choose $(\theta_H, \theta_H^2)$

2. If accepted by both types of buyers then $(q, t)$ must satisfy

$$\theta_L q - p \geq 0$$

$$\theta_H q - p \geq 0$$

   Due to complementarity, the first condition implies the second one. Hence, we need to check for the first condition only. Then, in order to $(q, p)$ to be optimal, it must solve

$$\begin{aligned} \max_{q,p} \quad & p - q^2/2 \\ \text{s.t.} \quad & \theta_L q - p \geq 0 \end{aligned}$$

   It is easy to see that the solution is then given by $(\theta_L, \theta_L^2)$.

Hence, only the full observability quality-price pairs will be offered.

In order to prove that it is optimal to offer only one of these two alternatives, we will consider a modified model in which the seller has only the two contracts described above available to choose from but he could determine the probability of each contract being drawn by buyers. We will assume this probability must be the same for all buyers (regardless of their valuation).

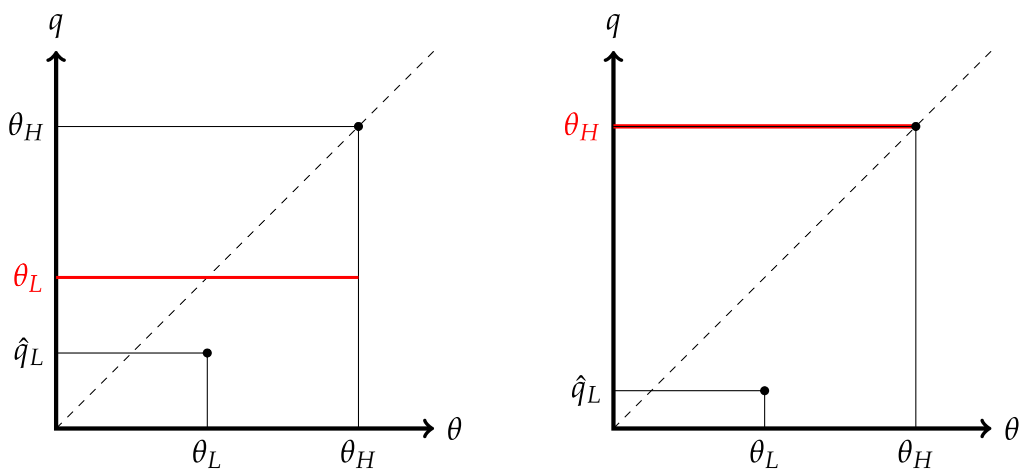
For this modified model, the seller's problem could be written as

$$\max_{x \in [0,1]} x\alpha(\theta_H - \theta_H^2/2) + (1 - x)(\theta_L - \theta_L^2/2)$$

where $x$ is the probability contract $(\theta_H, \theta_H^2)$ is drawn. Note that this problem is linear in $x$, hence either $x = 1$ or $x = 0$ would be optimal depending on the parameters of the model. Let $\hat{\alpha} = \left(\frac{\theta_L}{\theta_H}\right)^2$. Then we have that $x = 1$ would be optimal if $\alpha > \hat{\alpha}$, while $x = 0$ would be optimal if $\alpha < \hat{\alpha}$. □

Hence, in an environment with a single sample the *effective variety* is reduced. That is, only one combination of quality and price is offered to the buyers. However, this cannot be interpreted as the seller offering a simpler mechanism. Indeed, the seller could still have incentives to *obfuscate* the market by introducing several products with the same quality and price, making the actual menu offered more complex. Hence, we cannot rule out that the seller will include more than one alternative in the optimal menu but instead show that all offered alternatives must be essentially the same. What is rules out is the possibility of including quality-price pairs which are never accepted by the buyers since such alternatives are costly for the seller in terms of lost sales since the probability of drawing the relevant offers decreases.

Doval and Skreta (2022) also obtains reduced variety in a nonlinear pricing model, however the rationale behind their result is different. Doval and Skreta (2022) consider a two period model with fully rational buyers and limited commitment, and obtain that the seller reduces variety in the first period in order to commit himself on not using all the information in the second period. Instead here the reduction in product variety is a consequence of the costs introduced by the limitations on the rationality of the buyers.




(a) $\alpha < \hat{\alpha}$: optimal menu offers only $q = \theta_L$ (red). All buyers accept the offer.

(b) $\alpha > \hat{\alpha}$: optimal menu offers only $q = \theta_H$ (red). Only buyers with valuation $\theta_H$ accept the offer.

Figure 1: Optimal menu with a single sample

Figure 1 illustrates the quality provided in the optimal menu with a single sample (red) and in the optimal menu when the menu is observable (black). If the proportion of high valuation buyers is low, then the optimal menu offers only a low quality product with a price equal to gross utility that a buyer with low valuation obtains from such quality level. Since this offer is accepted by all buyers, the quality received by low valuation buyers increases in comparison to the case with an observable menu but reduces the quality for high valuations buyers which no longer receive the efficient quality level for their valuation. If the proportion of high valuation buyers is higher, then only a high quality version of the product is offered at a price equal to the gross utility of the high valuation buyers for that quality level. In this case, only high valuation buyers will accept the offer while low valuations buyers will be completely excluded. The effect on the quality provision further depends on how large is the proportion of high valuation buyers. For intermediate values of $\alpha$, low valuations buyers are excluded in this case while they were served if the menu was observable, hence the quality provided to such buyers is effectively reduced in this case, while for high values of $\alpha$ they were also excluded when the menu was observable so there is no change in the quality provision in this case. For high valuation buyers, as long as $\alpha > \hat{\alpha}$ they receive the efficient quality for their valuation level hence there is no difference in quality provision compared to the case in which the menu is observable.

# 4 Some results in the case of two samples

We now consider the sampling model with two valuations but allow buyers to obtains two draws from the menu designed by the seller. As we mentioned above, we will assume that sampling is with replacement, hence the seller will design the menu before buyers could interact with it, and after that she will lose any control over it (i.e., she cannot change offers, prices, etc).

Given a menu of size $m$, buyers will observe only alternative $i$ with probability $1/m^2$, and will observe two offers $i$ and $j$ simultaneously with probability $2/m^2$. Since, now the probability of observing more than one offer is positive (unless all offers in the menu are identical), incentive compatibility constraints will be relevant in this case.

**Lemma 1.** *Consider the problem with two samples. Suppose that the optimal menu contains only two offers $(q_a, p_a)$ and $(q_b, p_b)$. Then, the expected profits generated by each offer must be the same.*

*Proof.* Suppose the optimal menu contains only two offers $(q_a, p_a)$ and $(q_b, p_b)$ with $(q_a, p_a) \neq (q_b, p_b)$.

Let $v_a$ and $v_b$ be the expected profits generated if only offer $a$ or $b$ is observed respectively. Let also $v_{ab}$ be the expected profits generated when both alternatives are observed.

Let fix the offers $a$ and $b$ and consider the problem of deciding the probability each offer is drawn with. Let $x$ be the probability offer $a$ is drawn. Then, with $n > 1$ samples, only offer $(q_a, p_a)$ will be obtained with probability $x^n$, only $(q_b, p_b)$ with probability $(1-x)^n$, while both offers will be observed simultaneously with probability $(1 - x^n - (1-x)^n)$.

Given this, the problem the seller will need to solve could be written as

$$\max_{x\in[0,1]} x^n v_a + (1-x)^n v_b + (1 - x^n - (1-x)^n) v_{ab}$$

Note that having an interior solution requires $v_{ab} \geq \max\{v_a, v_b\}$, i.e., profits from the full menu being larger than profits from each individual offer. Hence, we will focus in the case in which $v_{ab} > \{v_a, v_b\}$ here.

The first order conditions is

$$nx^{n-1} - n(1-x)^{n-1} - \left(nx^{n-1} - n(1-x)^{n-1}\right) = 0$$

From which we get

$$x = \frac{1}{1 + \left(\frac{v_{ab}-v_a}{v_{ab}-v_b}\right)^{\frac{1}{n-1}}}$$

Hence, in order to the optimal menu to contain only one copy of $a$ and $b$, we need $x = \frac{1}{2}$, which for a fixed $n$, holds only if $v_a = v_b$. □

The intuition behind this result is as follows. Suppose $v_{ab} > v_a > v_b$, so the profits of the complete menu are larger than the profits obtained from any of the two offers in isolation. Then, starting with a menu that only contains offer $a$, allowing offer $b$ to be drawn with a positive but small probability $\epsilon$ increases seller's payoff since it is more likely to get the full menu than a menu with only option $b$ ($2\epsilon(1-\epsilon)$ and $\epsilon^2$ respectively). Where, having the complete menu is the benefit of including $b$ while having a menu with only $b$ is the cost of including $b$.

For $v_a = v_b$, there is no loss on including $b$ so it is optimal to maximize the probability of getting the full menu, i.e., $x = 1/2$.

Hence, this result give a condition that the optimal menu needs to satisfy in order to contain only two offers: that those offers generate the same expected profits.

Given this result we can show one of the features that the optimal menu will have.

**Proposition 2.** *Consider the problem with two samples and two valuations. Then, the optimal menu does not contain only two different offers.*

*Proof.* Suppose the optimal menu contains only two offers $(q_a, p_a)$ and $(q_b, p_b)$ with $(q_a, p_a) \neq (q_b, p_b)$.

In order to this menu to be optimal, it must induce self-selection when both offers are observed since otherwise a single offer would be optimal. Hence, without loss we can focus on menus in which $(q_a, p_a)$ is the offer preferred by low valuation buyers, while $(q_b, p_b)$ is the one preferred by high valuation buyers if both options are available.

Then, the optimal menu must solve

$$\max_{(q_a,p_a),(q_b,p_b)} \frac{1}{2}\left(\alpha\left(p_b-\frac{q_b^2}{2}\right)+(1-\alpha)\left(p_a-\frac{q_a^2}{2}\right)\right)+\frac{1}{4}\alpha\left(p_b-\frac{q_b^2}{2}\right)+\frac{1}{4}\left(p_a-\frac{q_a^2}{2}\right)$$

subject to $\theta_L q_B - p_b \geq 0$ and $\theta_H q_a - p_a \geq \theta_H q_b - p_b$.

We are interested only on the cases in which an interior solution exists. Hence, we will proceed with the analysis assuming that an interior solution exists.

The solution involves qualities $q_a = \theta_L - \left(\frac{3\alpha}{3-2\alpha}\right)(\theta_H-\theta_L)$ and $q_b = \theta_H$, with prices $p_a = \theta_L\left(\theta_L - \left(\frac{3\alpha}{3+\alpha}\right)(\theta_H-\theta_L)\right)$ and $p_n = \theta^2 - (\theta_H-\theta_L)\left(\theta_L - \left(\frac{3\alpha}{3+\alpha}\right)(\theta_H-\theta_L)\right)$. This coincides with solution of the standard problem where all alternatives are observed for a properly defined proportion of each type of buyer.

From Lemma 1, in order to this menu to be optimal we need

$$\theta_L\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)-\frac{\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)^2}{2}=\alpha\left(\frac{\theta_H^2}{2}-(\theta_H-\theta_L)\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)\right)$$

We define the function

$$\begin{aligned} F(\alpha) = \ & \alpha\left(\frac{\theta_H^2}{2}-(\theta_H-\theta_L)\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)\right) \\ & -\theta_L\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)-\frac{\left(\theta_L-\frac{3\alpha}{3-2\alpha}(\theta_H-\theta_L)\right)^2}{2}. \end{aligned}$$

The above condition is equivalent to compute the zeros of this function. Note that $F$ is a strictly increasing continuous function, $F(0) < 0$, and $F\left(\frac{3\theta_L}{3\theta_H-1}\right) > 0$, where $\frac{3\theta_L}{3\theta_H-1}$ is the value that makes $q_a = 0$. Hence, using the Intermediate Value Theorem, we have that there exist a unique value of $\alpha$ such that $F(\alpha) = 0$. So, having only two alternatives in the menu is optimal for a set of parameters with value zero in this case.

□

Note that while we ruled out the optimal menu to have only two offers, it could still contain a single offer since having a single offer is optimal under full observability in some cases. However, there could still be gains from offering more than one alternative as well. Indeed, as Lemma 1 having more than one copy of the offers could be optimal if the menu contain differentiated offers. A consequence of this is that, in general, the optimal menu would be more complex than the offering when the menu is observable: identical copies of a product could be included without aggregated value in order to reassure buyers will evaluate one of the products offered by the seller.

# 5 Extension: finite size sub-menus with a single sample

In this section we present an extension in which the seller offers a collection of menus from which the buyer obtains a single sample and then decide what product to purchase.

Consider an environment identical to the one presented before but in which the seller instead of offering single quality-price pairs to be sampled by the consumer, he can design small menus that the buyers could observe in their single draw. That is, the seller not only designs the quality-price pair but also some of the other products the buyer will actually compare too. Through this, the seller is better off than in the environment in which buyers only sample single quality-price pairs, but still weakly worse off than in the case with complete observability.

In order to make this extension interesting, we consider a setting in which the buyers valuation is distributed over the interval $[0, 1]$ following a distribution $F$ with a strictly positive density $f$ in $(0, 1)$. We assume there is a limit to the size each sub-menu could have, and denote it by $S$. We also assume there is a unique maximizer to the problem of choosing a single sub-menu of size $S$.

**Proposition 3.** *Consider the model with finite-size sub-menus. Then, in the optimal menu the seller always offers a single sub-menu.*

*Proof.* We will show that the optimal mechanism only contains a single menu, and that this menu should be the (unique) maximizer of the profits under a single-submenu.

Steps: (i) unique maximizer should be part of the optimal mechanism: if not, replacing any sub-menu with the unique maximizer increases expected profits; (ii) if there is another sub-menu then replacing it with the maximizer strictly increases the expected profits since it is the unique maximizer. □

# 6 Concluding remarks

We characterize the solution of a monopolistic screening problem with boundedly rational buyers which cannot observe the menu designed by the seller. Hence, instead of evaluating all alternatives offered by the seller they sample from the menu uniformly at random and then decide if accept one of the sampled offers if any. We show that the optimal menu if buyers sample only once is to offer a single quality-price pair, which matches the valuation of some buyers and charges exactly the value generated for that type of buyers.

In the case with two samples, we show that the optimal menu never involves using two different alternatives only, each drawn with probability $1/2$ but also can contain more than a single alternative.

Note that the seller in our setting is in a worse position than in the case in which the menu she offers is fully observable by the buyers. So, given the chance to improve buyers knowledge about the alternatives offered, she will try to do so. However, if noise persist is not necessarily true that the seller will simplify her offers and instead could strategically use obfuscating menus in order to obtain higher profits.

The model we presented here abstract from any form of direct targeting: all buyers face the same menu and have the same probability of observing each alternative. Since the use of advertising tailored to specific types of consumers is well spread in today's economy, a natural extension of our model would consider the impact of partially personalized menus into the design problem of the firm.

Another venue worth exploring is the effect of competition among firms in this setting.

While this paper studies only the nonlinear pricing model in Mussa and Rosen (1978), we think our model could be applied to more general settings in which the decision of agents are based on imperfect observation or evaluation of the alternatives available to them. For example, it could be applied to a setting in which an agent faces a complex tax system. So, instead of evaluating all the potential benefits he is able to apply for, he randomly observe only some of them and evaluate whether he is eligible to apply for such benefit. In this context, we think our model could provide a simple environment to analyze the impact of this behavior on the design of relevant policies.

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